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First Results from a Microeconomic Model

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The Development of Computer-Networks:

First Results from a Microeconomic Model

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Abstract

Computer networks like the Internet are gaining importance in social and economic life. The accelerating pace of the adoption of network technologies for business purposes is a rather recent phenomenon. Many applications are still in the early, sometimes even experimental phase. Nevertheless, it seems to be certain that networks will change the socioeconomic structures we know today. This is the background for our special interest in the development of networks, in the role of spatial factors influencing the formation of networks and consequences of networks on spatial structures, and in the role of externalities. This paper discusses a simple economic model – based on a microeconomic calculus – that incorporates the main factors that generate the growth of computer networks. The paper provides first analytic results about the generation of computer networks. The paper discusses (1) under what conditions economic factors will initiate the process of network formation, (2) the relationship between individual and social evaluation, and (3) the efficiency of a network that is generated based on economic mechanisms.

1. Introduction

The development of computer technologies and its applications has been extremely rapid in the eighties and nineties. Computer networks belong to the most significant fields of technical development. This is a consequence of two development paths:

1. The growing importance of PCs due to the rapidly increasing computing capacity.
2. The organizational and communicative advantages of electronic networks have led to the widespread use of private networks, not only within companies and its subsidiaries but also in customer-supplier-relations.

The main impulse came from a public network - the Internet. It is generally believed that computer networks like the Internet will fundamentally change economy and society (see e.g., Castells, 1996, MacLuhan, 1992, Gillespie, 1991). Castells (1996) even compares this new development with the Industrial Revolution.

In order to understand what consequences computer networks will have on society and economy it seems necessary to first understand why and how such networks develop. In this paper we try to take a step in this direction. We attempt to find key economic factors that may have driven the rapid international diffusion of networking over the past years, and to identify the conditions that may stimulate or hamper this development. We will do this by developing a conceptual framework and a simple economic model that captures the key factors.

In this paper we apply a microeconomic perspective. Rational agents are assumed to decide about whether to connect to another agent through a network link or not based on a comparison of their costs and benefits of this step. In order to keep the analysis manageable we use relatively simple concepts of costs and benefits in this context. Most importantly, we assume that all the costs and benefits materialize in the current period so that we don't have to take into account discounting and expectations about the future development of the network.

Of course, in reality there are different components of the costs of a network link: installation costs, fixed and variable telecommunication costs, provider fees, etc. In our analysis we assume only the costs of establishing the network link. A similar argument holds for benefits as well. Here we assume that the main effect of a computer network link lies in the reduction of distance friction. This, in turn, allows for an increased level of interaction between actors. The actors are assumed to derive utility from this interaction.

A major economic element of any network infrastructure lies in the existence of network externalities (Capello, 1994). In our simple model network externalities originate from the fact that when an actor connects to another actor who is already on a network, distance friction between all actors already on the network and the new member is reduced. As we will see, these network externalities are the main driving force behind network development. They also raise the question of optimality and individual vs. social valuation.

2. The Model

In order to gain some insight into the development of computer networks, we now develop a model that allows us to talk about the above mentioned mechanisms and relationships in more precise terms.

We start by describing the initial condition of the problem, i.e. the situation without any computer network at all, by a complete graph

$$G = (V, E) \tag{1}$$

with V being a set of n vertices and E the set of all $\frac{n(n-1)}{2}$ possible connections between these vertices. The weights of the graph are given by a weight – or distance – matrix **D**. Economic actors are located at the vertices of the graph G. This weight matrix represents the general distances between each pair of actors.

The computer network is described by another graph $N = (V, L)$ with the same set of vertices as G and a set of edges L that is a subset of E and represents existing computer links. Initially, we set $L_0 = \{\}$ and denote this graph as $N_0 = (V, L_0)$.

The graph N may consist of a number of components, i.e. the subgraphs formed of connected vertices and the corresponding edges. Since unconnected vertices are defined as components, the number of components may be between 1 (when all vertices are connected by the same network) and n (when no network connections exist). We will denote the components of graph N as

$$c_i(N),\ i = 1, \ldots, C(N) \tag{2}$$

where $C(N)$ is the number of components of graph N.

With these definitions, each vertex must belong to exactly one of the components of N. We will write the component that contains vertex V_i as c^i. Note that when V_i and V_j are directly or indirectly connected by a network link, i.e. by network links in L, they belong to the same component and $c^i = c^j$.

The actors are assumed to benefit from interaction. They can either interact along the edges of the underlying graph G, or along the network links in N. Of course, they can interact along N only with those actors that belong to the same component; i.e. those who are on the same physical network.

Network links ease interaction and are therefore beneficial for the actors. We define the benefit of interaction as a standard interaction potential. For N_0, the case with no network links, the benefit for actor i is therefore

$$B_i(N_0) = \sum_j \exp(\alpha D_{ij}) \tag{3}$$

Now, what is the effect of a network link on the benefits of a certain actor? It can be argued that in today's computer networks distance friction is negligible. Delays in the communication over those networks originate from other factors like limited capacity of a server, limited capacity of the final link, a general level of congestion, etc. that are not related to distance. Therefore, it can be argued that distance friction is completely eliminated by network links. The benefits for an actor on the network would therefore become

$$B_i(N) = \sum_{j \notin c^i} \exp(\alpha D_{ij}) \quad + \quad \sum_{j \in c^i} \exp(0) \tag{4}$$

This, of course, is an extreme position that may hold only for pure communication. As soon as there are other forms of interaction involved, like the shipment of products or personal visitations, distance friction is again important. Therefore, a more moderate formulation would be to allow the network links to reduce distance friction by a certain factor δ $(0 \le \delta \le 1)$. The benefits for an actor would therefore become

$$B_i(N) = \sum_{j \notin c^i} \exp(\alpha D_{ij}) \quad + \quad \sum_{j \in c^i} \exp[\alpha(1-\delta) D_{ij}]. \tag{5}$$

This is the most general formulation. It contains the two others as special cases *(δ = 0* and δ = *1*, respectively).

For any $\delta > 0$ the actors involved benefits from a network connection. But, of course, there may also be costs involved in establishing and operating a network link. These costs are probably proportional to the length of the network link. If we denote a direct network connection between vertex i and vertex j as L_i^j, the costs for this link can be written as

$$C(L_i^j) = \gamma D_{ij}. \tag{6}$$

Note that with the establishment of a link from *i* to *j* we also establish a link from *j* to *i*. Therefore, the question arises, who will bear the costs of this network link. In the worst case, one of the two actors that set up a new network link will have to bear all the costs. Therefore, in his/her decision making every actor will have to assume that he/she will have to bear all the costs of the network link. Consequently, we can write for $C_i(L_i^j)$, the costs the link may create for actor *i* as

$$C_i(L_i^j) = C(L_i^j) = \gamma D_{ij}. \tag{7}$$

Similarly, $C_i(N)$ is the costs network *N* generates for actor *i*.

The question of our paper is, how *N* evolves over time, when the decisions about whether to establish a certain network link or not are based upon the economic calculus of the actors.

For each pair of graphs *G* and *N* we can compute the benefits and costs for each actor. This yields vectors of benefits and costs

$$\mathbf{B} = B(N,G) \quad \text{and} \quad \mathbf{C} = C(N,G). \tag{8}$$

Since *G* is a constant, we will use the simplified notation

$$\mathbf{B} = B(N); \quad \mathbf{C} = C(N). \tag{9}$$

Given these benefits and costs, the actors are, of course, interested in maximizing their profits:

$$\Pi_i(N) = B_i(N) - C_i(N) \tag{10}$$

As mentioned above, we assume throughout that all the costs and benefits occur only in the current period. That means that actors look only at the current situation and do not make any investment-type decisions where they trade off current expenditures for future benefits. Also, we assume that there are no capacity constraints. Therefore, the addition of a network connection cannot lower the benefit of actors already on the network. These two assumptions simplify the problem considerably, but at the same time severely limit the potential value of the analysis. We hope to be able to remove these assumptions in future work.

Let us write as $N_i^{+j} = (V, L + L_i^j)$ the graph that we get when adding the network connection L_i^j to a base graph *N*. Similarly, $N_i^{-j} = (V, L - L_i^j)$ is the graph that we get when we remove the network connection L_i^j from the base graph N. But, since we do not allow for capacity constraints, network links will only be added in our analysis, never removed.

At every time period each actor can choose from *n* different strategies. She can stay with the current situation, drop the network link to any of the – say *k* – other actors he/she is currently connected to, or establish a link to any one of the *n-k-1* actors she is currently not connected to.

The marginal profits of these strategies can be derived directly from the above definitions:

$$\Pi_i(N_i^{+j}) - \Pi_i(N) = [B_i(N_i^{+j}) - B_i(N)] - [C_i(N_i^{+j}) - C_i(N)] \quad (11)$$

$$\Pi_i(N_i^{-j}) - \Pi_i(N) = [B_i(N_i^{-j}) - B_i(N)] - [C_i(N_i^{-j}) - C_i(N)] \quad (12)$$

The marginal profit of no change at all is zero, of course.

At every time period a rational actor will try to implement the strategy that provides him/her the maximum marginal profit. However, implementing such a strategy has implications for other actors as well. It can well be that adding link L_i^j yields the highest marginal profit for actor *i*, but does not yield the highest marginal profit for actor *j*. A number of mechanisms are conceivable that decide such situations. For example, we may assume that only the strategy with the highest marginal profit of all actors will be implemented. The most meaningful mechanism from a microeconomic point of few is what we call the „mutual best" strategy. This means that only those network links will be established that yield the highest marginal profit for both actors involved.

3. Some Results

3.1. Network Generation

We can derive some interesting first results even from this very simple structure. The first question we will ask is, under what conditions the economic forces will be sufficient to generate a network. More formally: Under what conditions will economically rational actors establish at least one network link. A related question that we can answer with the first one is which one of the possible connections will be established first.

From (5) we know that the following holds:

$$B_i(N_{0i}^{+j}) - B_i(N_0) = \exp[\alpha(1-\delta)D_{ij}] - \exp(\alpha D_{ij}) \quad (13)$$

The marginal costs for actor i are simply a linear function of distance:

$$C_i(N_{0i}^{+j}) - C_i(N_0) = \gamma D_{ij} \quad (14)$$

When we plot the marginal benefits as a function of distance we see that for $0 < \delta < 1$ the function is positive, but approaches zero as distance increases. The maximum of the function is where

$$D_{ij} = \frac{-1}{\alpha\delta} \ln(\frac{1}{1-\delta}) . \tag{15}$$

For $\delta = 1$, i.e. the case where distance friction is eliminated completely, the marginal benefit increases monotonically and approaches 1. Since $\delta = 0$ represents the case when distance friction is not changed at all by network links, the marginal benefit is always zero in this case.

Since the marginal benefit is positive for all meaningful values of δ, the answer to the question we have posed depends upon the slope of the cost function. When the slope of the cost function is lower than that of the marginal benefit at distance zero, there will be a range of distances where the creation of a network link is beneficial. Since the slope of the marginal benefit function at distance zero is $-\alpha\delta$, we find that only when

$$-\alpha\delta > \gamma \tag{16}$$

a network link may be created by rational economic agents.

When this condition holds, the range of distances at which establishing a network link is economical always begins at distance zero. For $\delta < 1$ and $\gamma > 0$ there must be a maximum distance beyond which the marginal profit is negative. The „optimum distance“, the distance for which the marginal profit is highest, must be somewhere between zero and this upper bound.

Because of the difference of exponential functions in (5) we cannot calculate an exact solution for the upper bound and the optimum distance. When we approximate the benefit function by a Taylor series expansion, we find that the upper bound and the optimum distance are approximately

$$\frac{2(\gamma+\alpha\delta)}{\alpha^2\delta(\delta-2)} \quad \text{and} \quad \frac{(\gamma+\alpha\delta)}{\alpha^2\delta(\delta-2)} . \tag{17}$$

When the parameters are such that for a certain range of distances in **D** positive marginal profits exist, will there always be a network link established? Under our set of assumptions the answer is yes. The reason is that the network link that yields the highest marginal profit of all possible network links must also yield the highest marginal profit for the actors at both its ends. Therefore, whenever at least one distance exists that provides a positive marginal profit, a network link will be established for economic reasons.

Because of (16) and the condition that δ is at most equal one, both nominators and the denominator in (17) are less than zero. Therefore, when γ increases, the optimum distance and the upper bound decrease. This implies, for example, that in a country where telecommunication costs are high, computer networks will start off with shorter connections than in a country where telecommunication costs are low.

The argument that we have made so far also holds when network links can be established for free ($\gamma = 0$). Although the marginal profit is positive for all distances, the maximum marginal profit is at the distance given by (15) above. So, even when network links are free, the network will begin with that connection that is closest in distance to the optimum distance given in (15). The reason is that interaction partners that are further away than this optimum distance contribute less to the benefit of the actor. Only when in addition to free network links ($\gamma = 0$) also distance friction is perfectly eliminated ($\delta = 1$), then actors will want to start the network by establishing the longest possible network link.

3.2. Individual vs. Social Profit

The next question we can discuss is that of the individual evaluation in relation to a social one. We can pose this question in the context of the above discussed problem and ask ourselves, whether there might be situations when based on the individual evaluation no first network link will be created although it would be desirable from a social point of view.

A difference between individual and social evaluation may result from two sources:

1. There might be benefits to other actors that are not taken into account by the decision maker, and

2. the costs to society may differ from those that the decision maker takes into account.

As mentioned above, both sources exist in our problem. Establishing a network link from *i* to *j* not only improves the interaction potential of actor *i*, but also that of *j* and all the other actors that are already connected to *i* or *j*. As far as costs are concerned, we have argued above that a-priori it is unclear who will have to bear the costs of a network link and that therefore each actor should base her decision upon the assumption that she will have to bear all the costs by herself. As stated in (7), the individual costs of an additional link are equal to the social costs. Therefore, on each side the marginal benefit of the network link must be higher than the total marginal costs. From a social point of view, however, the marginal benefits to all actors together have to exceed the marginal costs of the link.

We can derive the social benefit of a certain network *N* directly from (5) above by summing the benefits for all actors in the system. So, the social benefit can be written as:

$$B(N) = \sum_j B_j(N) \tag{18}$$

This allows us to derive a direct measure of the network externality at the benefit side, which is the sum of all the benefits that are not been taken into account by the decision maker. When *i* is the decision maker, the network externality is

$$B(N) - B_i(N) = \sum_{j \neq i} B_j(N) \tag{19}$$

Based on these arguments, we can derive the social marginal profit of the first network link between *i* and *j* as:

$$\Pi(N_{0_i}^{+j}) - \Pi(N_0) = \sum_k [B_k(N_{0_i}^{+j}) - B_k(N_0)] - [C_k(N_{0_i}^{+j}) - C_k(N_0)] \tag{20}$$

$$= 2\{\exp[\alpha(1-\delta)D_{ij}] - \exp(\alpha D_{ij})\} - \gamma D_{ij} \tag{21}$$

When we compare (21) to (13) and (14), we see that the social marginal profit differs from the individual one by the individual marginal benefit of the link. This is the benefit that actor *j* enjoys, but which is not considered by actor *i* in the decision making.

When we do the same calculations as above, we find that the equivalent to (16), the condition that a network link may be socially desirable, is

$$-2\alpha\delta > \gamma. \tag{22}$$

Obviously, the costs for the network link could be twice as high when social costs and benefits are taken into account than in the case of an individual calculation. So, when

$$-2\alpha\delta > \gamma > -\alpha\delta \tag{23}$$

a socially desirable network link may be possible, but will not be created based on the actors' microeconomic calculus.

We can also derive approximate solutions for the upper bound of the distance range and the optimum distance under a social calculus. They turn out to be

$$\frac{(\gamma + 2\alpha\delta)}{\alpha^2\delta(\delta - 2)} \quad \text{and} \quad \frac{(\gamma + 2\alpha\delta)}{2\alpha^2\delta(\delta - 2)}. \tag{24}$$

When we compare these results to (17), we find that both the upper bound and the optimum distance is higher under social evaluation. So, from a social point of view, the range of distances for which a network link may be created is too narrow, and when a network link is created based on the individual calculus, it will tend to be too short from a social point of view.

3.3. Network Structure

Another question we may ask is, whether a network that emerges step by step from this microeconomic mechanism will be efficiently serving the system of actors. In order to answer this question, we will have to look at the end point of the process. Suppose that the spatial constellation and parameter values are such that based on the decision process in the end all actors are connected to the network. Let us denote this network as N_n. It is easy to see that irrespective of how the actors are actually linked, this network will yield a vector of benefits with

$$B_i(N_n) = \sum_j \exp[\alpha(1-\delta)D_{ij}]. \tag{25}$$

The social benefit is

$$B(N_n) = \sum_i \sum_j \exp[\alpha(1-\delta)D_{ij}]. \tag{26}$$

The profit of the network therefore depends only upon the costs. The network N_n will be optimal when the actors are connected such that the sum of the costs of these connections is minimal. In graph theoretic terms this means that the vertices V must be connected by a „minimum-weight spanning tree“ (see e.g. Gibbons, 1985). Prim (1957) has shown that a minimum-weight spanning tree can be constructed by always finding the shortest distance between a vertex in the tree and one not in the tree and connecting the two. So, when we want to construct a network that serves the system at least costs, we should begin by selecting an arbitrary vertex *i* and connect it to that vertex *j* which is nearest to it.

This provides the basis for answering the question we have raised above. As we have seen, neither based on the individual nor based on the social calculus will *i* want to establish a network link to her nearest neighbor. Instead, she will want to establish a link to that actor that is nearest to the optimum distance, where the latter differs from zero when there exists a range of distances for which a network link is desirable. So, the network that emerges from a microeconomic calculus – irrespective of whether it is based on individual or social costs and benefits – will not connect the actors at least costs. There might always exist another set of connections that yields the same level of benefits at lower costs. As some experimentation shows, the mutual best mechanism that we have discussed may not even connect the actors by a tree. The resulting network may even display loops.

4. Conclusions

This paper represents a first step toward an economic analysis of the question of the formation of computer networks. Stimulated by the tremendous growth of the Internet, we try to investigate which economic factors drive this development toward global connectivity and how these factors influence the process of network formation.

In a first part (section 2) we develop a formal model of the key economic mechanisms that drive network formation. The model focuses on individual microeconomic decisions based upon a comparison of costs of network links and benefits in the form of an increase in the interaction potential. The model is still fairly simple, and there is room for improvement. For example, the model does not take into account capacity constraints, differences between actors, different decision strategies, etc.

In section 3 we analyze the behavior of this model and the implications it has for the process of network formation. We show

- under what conditions the economic incentive is sufficient to initiate the process of network formation,
- that there might be constellations where a network would be socially desirable, but the economic incentives are insufficient to generate any network links, and
- that the economic process of network formation does not lead to an optimal network topology.

As has been mentioned above, we view this paper as a first step in the direction of the topic and intend to develop more elaborated versions of the model later. Because of the complex relationships between the various actors in the process of network formation and because of network externalities and capacity constraints, we will probably have to use simulation experiments when analyzing these more complex model versions.

References

Capello, Roberta, 1994: Spatial Economic Analysis of Telecommunications Network Externalities. Avebury

Castells, Manuel, 1996: The Rise of the Network Society. Blackwell

Gibbons, Alan, 1985. Algorithmic Graph Theory, Cambridge, Cambridge University Press

Gillespie, Andrew: Advanced communications networks, territorial integration and local development. In: Camagni, Roberto (ed.), 1991: Innovation networks - spatial perspectives. Belhaven Press; p. 214 - 229

MacLuhan, Marshall and Powers, Bruce R., 1992: The global village - transformations in world life and media in the 21st century. Oxford University Press

Prim, R. C., 1957. Shortest connection networks and some generalizations, Bell System Tech. Journal, Vol. 36, pp. 1389-1401.

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