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Discrete Choice, Optimal Search

and Spatial Interaction Models:

Some Fundamental Relationships

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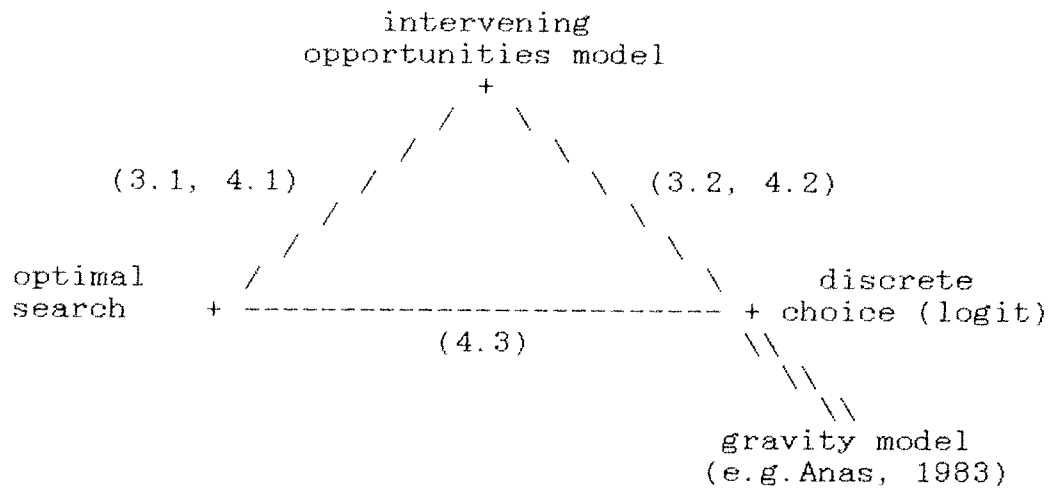   York at Buffalo, Buffalo, New York, USA

1. Introduction

The gravity model and the intervening opportunities model have long been the two most dominant methods for modeling spatial interaction. Both are macro models that are concerned with the magnitude of aggregate interaction between two points in space rather than the behavior of individuals. In contrast, the development of discrete choice models has come about as a consequence of recent emphasis on individual behavior. These models have found widespread application in the study of not only travel demand (e.g., Ben-Akiva Lerman, 1985), but also in geography and regional science more generally (e.g., Anas, 1983, Wrigley, 1985). The theory of optimal search provides an alternative framework for the study of individual choice behavior.

Surprisingly little use has been made of the theories of optimal search and discrete choice in the context of spatial interaction. This paper investigates the relationships between aggregate models of spatial interaction and behavioral models of search and choice. Since it is already well known that the most prominent version of discrete choice models, the multinomial logit model, is formally equivalent to the production-constrained version of the gravity model (e.g. see Anas, 1983), we will focus upon the links between optimal search, discrete choice models, and the intervening opportunities model (Figure 1).

The equivalence between the multinomial logit and the gravity model provides a useful connection between interaction at an aggregate level and the optimizing behavior of individuals. Just as the multinomial logit model gives a behavioral interpretation to the gravity model, the models discussed in this paper

1

Figure 1: Relationships Between Models of Discrete Choice, Optimal Search, and Spatial Interaction

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                        intervening
                    opportunities model
                            +
                          /   \
                         /     \
        (3.1, 4.1)      /       \    (3.2, 4.2)
                       /         \
                      /           \
    optimal          /             \        discrete
    search      + ------------------- +  choice (logit)
                        (4.3)          \\
                                        \\
                                         \\
                                  gravity model
                                  (e.g.Anas, 1983)
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Note: The numbers refer to the sections of the paper.

give a behavioral interpretation to the intervening opportunities model. A fundamental difference in this comparison is that while the logit and gravity models have been shown to be equivalent, the standard form of the intervening opportunities model is derived only from special cases of optimal search and discrete choice models. However, both discrete choice models and optimal search theory may be employed in a diverse range of behavioral settings. This diversity allows generalization of the rather restrictive form of the intervening opportunities model.

In the second section, the intervening opportunities model is reviewed, and some of its weaknesses are emphasized. In the third section, the intervening opportunities model is derived as a special case of both discrete choice models and optimal search

theory. Then in the fourth section, the generality of both discrete choice and optimal search approaches is employed to demonstrate the wide variety of spatial interaction models that may be derived.

Discrete choice models and optimal search theory are typically used in quite different situations. In general, discrete choice models are most frequently used in applied problems of simultaneous choice, while optimal search is more frequently used in understanding and conceptualizing the nature of sequential choice. Though the discrete choice and optimal search approaches are therefore quite different, there are important similarities as well. Relationships indicating the similarity of these two approaches used for the generalization of the intervening opportunities model are also discussed. The final section provides a summary and suggests a number of directions for future research.

2. The Intervening Opportunities Model

The intervening opportunities model was first developed by Stouffer (1940) to explain migration between origins and destinations. A more common form of the model is due to Schneider (1959), who used the concepts developed by Stouffer in the context of trip interaction. It is this latter version of the model that is now briefly summarized.

The probability of traveling beyond the first D_j opportunities is equal to $(1-L)^{D_j}$, where L is the constant probability of accepting an individual opportunity. The probability of stopping somewhere in the next zone k away from the origin is then:

$$p(k) = (1-L)^{D_j} - (1-L)^{D_j+D_k} , \tag{1}$$

where D_k is the number of opportunities in zone k. Linearization of the binomial expansion with respect to L on the right-hand side of (1) allows a continuous version to be derived:

$$p(k) = b \left[e^{-LD_j} - e^{-L(D_j+D_k)} \right] ,\qquad\qquad (2)$$

where the constant b may be chosen to ensure that the probabilities sum to one over all alternatives (Wilson, 1970). Schmitt and Greene (1978) provide an alternative derivation of (2) that avoids the linear approximation described above.

The intervening opportunities model has been extended in a number of directions. Kitamura (1985), for example, combines concepts of utility maximization and trip chaining to construct an intervening opportunity model for a linear city. Relationships between the intervening opportunities model and other lines of research have also been noted. Okabe (1976), for example, noted the conditions under which the gravity and intervening opportunities model give rise to approximately equivalent trip patterns. Weibull (1978) was apparently the first to suggest that the intervening opportunity model may be regarded as a special case within a more general search-theoretic framework. In the following sections, we elaborate on this latter suggestion, and draw a number of other connections to discrete choice theory.

We wish to specifically address two striking weaknesses of the intervening opportunities model. First, the traditional form of the model is extremely inflexible, owing to the assumption of a constant probability of acceptance, L. This restriction implies a geometric decline in the probability of interacting with more distant opportunities in the discrete formulation, and an exponential decline in the continuous one. Another criticism of the intervening opportunities model is that the implied

behavior of individuals is overly simplistic. Individuals obey the principal of least effort by stopping at the first acceptable opportunity that they examine, but this is the only type of explicit behavior displayed. There is no particular concern with the specific value of the attribute(s) received at the destination selected. Thus the model is one where individuals exhibit "satisficing" behavior, with the further restriction that every opportunity is equally likely to be satisfying. In the next section, the intervening opportunities model is derived as a special case of more general sequential search models and discrete choice models. We will show that intervening opportunities models may be generalized by relaxing the restrictive assumption of a constant probability of acceptance, and also by adding an individual concern with the value of the attribute(s) received. Moreover, these models are based on the assumption of optimizing rather than satisficing behavior.

3.   Optimal Search, Discrete Choice and the Intervening Opportunities Model

Optimal stopping theory (DeGroot, 1970; Chow, Robbins, and Sigmund, 1971) provides a very general framework for capturing individual behavior in a wide variety of settings involving sequential decisions. The theory has been applied to many diverse problems, including the selling of housing and other assets (Albright, 1977; Stull, 1978; Rosenfield et.al., 1983; Rogerson, 1985). Decisions are also often made sequentially when buying assets, and hence the theory of optimal stopping should prove useful in modeling certain spatial interaction problems, such as shopping trips.

5

Discrete choice theory assumes a simultaneous decision process. Individuals are assumed to face a set of discrete alternatives and choose the one that maximizes their utility (Domencich and McFadden, 1975; Hensher and Johnson, 1981; Maddala, 1983; Ben-Akiva and Lerman, 1985; Wrigley, 1985). The theory has been used extensively during the last decade. Among others applications it has been used to model transport behavior (Domencich and McFadden, 1975), brand choice (Louviere and Hensher, 1983), migration (Bartel, 1979), and labor supply (Long and Jones, 1980).

Either of two scenarios are relevant here. In the first, the destinations have deterministic values of attractiveness, and the stochasticity is introduced via the researcher's limited ability to observe all of the relevant characteristics and the heterogenous tastes and preferences of the many individuals choosing destinations. That is, the utility actually derived by different individuals will vary due to observation errors and heterogeneity in tastes and preferences. The variation in preferences may be modeled by a probability distribution describing the different utilities received at a particular destination by different individuals. Alternatively the focus may be on the behavior of a single individual facing a probability distribution that governs the stochastic attractiveness of destinations. Discrete choice theory usually applies the first scenario (see e.g. Hensher and Johnson, 1981; Anas, 1983), while optimal search theory arguments are phrased in terms of the latter one. Thus, in this paper we use both concepts.

When dealing with spatial choice processes, three types of costs need to be distinguished:

- <u>Intrinsic costs</u> are a characteristic of the alternative (price, cost of shipment, etc.). They are paid only at the destination actually chosen.

- <u>Search costs proper</u> are part of the costs for investigating a destination. They are paid for each investigated destination and have no effect on the costs of other destinations. Examples are the time and effort necessary for checking a destination.

- <u>Travel costs</u> are also paid for each investigated destination but bearing them for one reduces the costs of some other by the same amount.

To illustrate, suppose an individual is driving down a road to buy some furniture, say a chair. There are a number of furniture stores along the road. The individual stops at the first one, and checks whether they have the type of chair he is looking for and asks for the price. Clearly the price of the chair belongs to the first category, <u>intrinsic costs</u>. He has to pay only if he decides to buy the chair. The time and effort it takes to park the car, walk into the store and ask for a chair and its price are <u>search costs proper</u>. When we ignore the possibility of learning, there are no benefits from these costs when the chair is not purchased. The costs of traveling to the first store are part of the third category, <u>travel costs</u>. Traveling to the first store brings the individual closer to other stores as well, and thus reduces the extra costs for searching among these stores.

Travel costs reflect the spatial distribution of opportunities and are therefore of particular importance in a spatial context. However, the term "travel costs" does not mean that

transport costs always belong to this category.   When our custo-
mer cannot go directly from one store to another but always has
to return to the starting point, transport costs belong to search
costs proper.

Travel costs and search costs proper can be viewed and
modeled as marginal or as total costs. Marginal travel costs are
the costs of proceeding to the next destination, total travel
costs of search are the costs of traveling from the origin to the
alternative examined.   Search costs proper as defined above are
marginal costs.   Contrary to travel costs the corresponding total
search costs proper cannot be derived generally, since they
depend on the sequence in which the opportunities are searched.

Figure 2:   The Basic Structure of the Intervening Opportunities
         Model
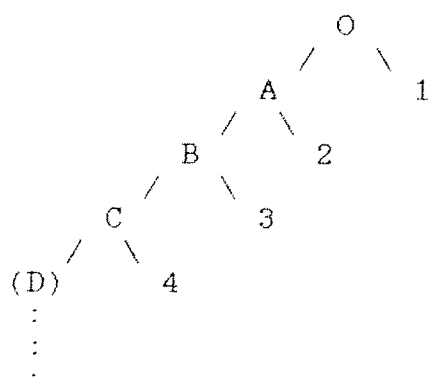
3.1. Sequential Optimization and the Intervening Opportunities
   Model

The basic structure of the intervening opportunities model
may be sketched as in Figure 2. The probability that an indivi-
dual starting at O ends up at opportunity 1 is assumed to be L.
If opportunity 1 is not chosen, the individual proceeds to A and
by assumption chooses opportunity 2 again with probability L -
conditional on the event that opportunity 1 was not chosen. The
same rationale applies for the lower levels as well and yields
the model discussed in section 2. In the case of a limited number
of opportunities, the selection probabilities are usually re-
scaled to sum to one.

The same basic structure applies to optimal search models.
They utilize the second concept of stochasticity mentioned above.
Suppose that a risk-neutral, utility-maximizing individual knows
the distribution of opportunities and searches them without
recall. At each node (O, A, B, C) the individual can draw one
opportunity, evaluate it, and decide either to accept it and stop
or to reject it and continue search. Since by assumption the
individual does not know the values of the following opportuni-
ties his optimal strategy is to accept opportunity 1 if its value
is greater than the expected return from continued search. Thus,
the probability of selecting opportunity 1, given that the
individual searches at all, is

$$P(1|O) = P(x_1 > y_1) \, , \tag{3}$$

where x_1 indicates the value of the first opportunity, y_1 the
expected return of continued search, i.e. the return the indivi-
dual can expect from proceeding to node A. Assuming both to be
defined in terms of money y_1 is defined as:

$$y_1 = -c_2 + \text{Emax } (x_2, y_2) \ , \tag{4}$$

where c_2 is the cost of observing the second opportunity consisting of marginal search costs proper and marginal travel costs of search. The conditional probabilities of selecting other opportunities, given that the individual has reached the node before (e.g. node A in the case of opportunity 2) can be found in an analogous way.

In general the conditional probabilities assume different values. However, under quite restrictive conditions, one obtains the same conditional probability for each opportunity and thus a model which is equivalent to the intervening opportunities model. These conditions are an infinite number of opportunities, all opportunities having identical search cost and their values being independent identically distributed.

In this case the expected return of continued search at all levels can be obtained from

$$c = \int_y^\infty (x-y) dF(x), \tag{5}$$

and the conditional probabilities are

$$L = 1 - F(y) \tag{6}$$

Thus, the intervening opportunities model can be viewed as a search model with an unlimited number of opportunities, identical search cost and an independent identical distribution of opportunities, which is known to the risk-neutral individual.

3.2. Simultaneous Choice and the Intervening Opportunities Model

Alternatively, the basic structure of the intervening opportunities model (Figure 2) can be interpreted as a nested discrete choice model. In this case we utilize the first concept of stochasticity mentioned above.

Although nested models are usually less restrictive than simultaneous discrete-choice-models, in this section we only use the nested equivalent of the simultaneous model. Since stochasticity is introduced at the level of the researcher the individual can choose among the opportunities simultaneously and thus will not face search costs proper. We observe the searcher choosing opportunity 1 if its value (x_1) is greater than the maximum value of the opportunities at the lower levels (z_1):

$$P(1|0) = P(x_1 > z_1) \ . \tag{7}$$

As in the search model, we assume that there are marginal travel costs of search when going from one opportunity to the next. z_1 can be written as:

$$z_1 = -c_2 + \max (x_2, z_2) \tag{8}$$

Note that z is a random variable rather than an expected value as y in the search model. This is the principal difference between the two models; it results from the difference between simultaneous and sequential decisions.

The conditional probabilities at the lower levels can be found in an analogous way and in general they will assume different values. This nested model, which is a sequence of binary choice models, is equivalent to a multinomial simultaneous model. In the case sketched in Figure 2 the opportunities of this multinomial model have values $(x_1, \ x_2 - c_2, \ x_3 - c_2 - c_3, \ x_4 - c_2 - c_3 - c_4)$. Note that we add the marginal costs to derive total travel costs of search. Identical conditional probabilities and thus a model equivalent to the intervening opportunities model can be derived under the same restrictive conditions as used for the search model: an infinite number of opportunities all having the same

search cost and being independent identically distributed. In this case z assumes the same value at each level:

$$z = -c + \max(x, x-c, x-2c, x-3c, \ldots) \ . \qquad (9)$$

Application of the distributional assumptions of the logit model, namely x being iid Gumbel distributed with location parameter θ and spread parameter μ (see e.g. Johnson and Kotz, 1970), yields the following choice probability for the kth opportunity:

$$p(k) = e^{-(k-1)\mu c} - e^{-k\mu c} \ . \qquad (10)$$

Note that this is very similar to equation (2) although the latter one is derived from an approximation. The conditional probability L becomes

$$L = 1 - e^{-\mu c} \ . \qquad (11)$$

The random variable z itself is iid Gumbel distributed with spread parameter μ and location parameter

$$\theta_z = -c + \theta - (1/\mu) * \log(1 - e^{-\mu c}) \ , \qquad (12)$$

which in the nested logit model is known as "inclusive value".

Thus the intervening opportunities model is equivalent to a highly restrictive version of a discrete choice model as well.

4.   Generalizations of the Intervening Opportunities Model.

The considerable flexibility of both the discrete choice and optimal search literature allow many generalizations of the standard form of the intervening opportunities model, which was derived as a special case of these frameworks in the previous section. In the first two subsections, we address ourselves to generalizations arising from optimal search and discrete choice approaches, respectively. In section 4.3, we explore the relationships between discrete choice and optimal search models.

4.1 Generalizations Derivable From a Sequential Search Approach

The story told in the previous section to arrive at the intervening opportunities model was a very special one. The search horizon was infinite, individuals had perfect information about the distribution of destination attributes, the cost of sampling each destination was constant, and individuals wished to maximize expected utility. In many situations, one or more of these presumed conditions may not hold. Consequently, it is desirable to have at one's disposal alternative models that adequately represent the actual sequential decision process. We now turn to a discussion of several of these alternatives.

A fundamental consideration in the formulation of an appropriate sequential decision model is the amount of destination information available to individuals. For convenience, we may classify sequential problems into categories where information is full, partial, or absent. In the basic search model used in the previous section, complete information on the parameters describing the attribute distribution was assumed. At the other extreme, no information at all may be available, and learning must take place as observations are taken. In intermediate situations, some a priori information is available, and knowledge about the distribution improves with additional observations.

4.1.1 No A Priori Destination Information

When no information about the functional form of the attribute distribution is available, objectives other than expected utility maximization must be considered (Stull, 1978). This is because expected utility can not be calculated without some

underlying assumption regarding the likelihood of alternative outcomes.

Gilbert and Mosteller (1966) describe the optimal strategy for individuals that wish to maximize the probability of obtaining the best alternative. Although the probability of stopping at the best destination is maximized in this formulation, the probability of stopping at a poor destination is also relatively high (Rogerson, 1986). Chow et al. (1964) consider the case where individuals adopt the less risky strategy of minimizing the expected rank of the attribute obtained at the selected destination, where destinations are ranked from best (rank 1) to worst (rank n). Although the probability of obtaining the best alternative is lower, the probability of obtaining "good", but nonoptimal values is significantly higher in comparison with the previous strategy.

Rogerson (1986) shows that of the two objectives described above, the rank minimization strategy generally leads to higher expected values and shorter waiting times before a choice is made. The rank minimization strategy will therefore exhibit a stronger distance decay pattern than will the interaction pattern of individuals maximizing the probability of obtaining the best destination.

4.1.2 Intermediate Information Available

In both of the previous strategies, optimal stopping leads to a lack of interaction close to the origin. While this characteristic is indeed what one should expect for the no information situation, few actual interaction matrices display this type of pattern. In most situations it will be more realistic to assume

14

that individuals have some prior information about destinations, so that they need not automatically pass up early choices simply to gain some initial information. Campbell and Samuels (1982) Petruccelli (1980) and Maier (1985) describe intermediate information scenarios that lie between the cases of no information and full information. In theory, it is possible to treat the amount of a priori information as a parameter, allowing a better fit between model and data.

4.1.3 Known Distribution of Destination Attributes

When the distribution of destination attributes is known, it is then possible to calculate the expected values of the attribute received. Under the usual assumptions of no recall of previous opportunities and a finite and known number of alternatives, the optimal strategy for individuals maximizing the expected attribute value is to first calculate an optimal decision number for each destination. A destination is selected if its attribute exceeds the relevant decision number. Optimal decision numbers (also known as reservation values) decrease as search continues, reflecting a searcher's tendency to grow less "choosy" as the number of alternatives diminishes. This is perhaps the most widely employed form of sequential search model; applications abound in studies of both the labor market and the housing market (Lippman and McCall, 1976; Smith et al., 1979).

Of course individuals could still employ other objectives. Rather than maximize the expected value of the attribute received, individuals who are more risk prone could still maximize the probability of selecting the best destination. This would lead to more search and lower average attribute values, but there

would of course be a greater likelihood of choosing the best destination. In general, a higher degree of risk aversion will lead to less search and earlier stopping.

4.1.4 Recall of Previously Examined Opportunities

It is in most instances more realistic to assume that there is a positive probability of selecting a previously examined opportunity. In this subsection we show how the logit choice model may be derived as a special case of a more general sequential decision model with recall.

When individuals have complete information about the distribution of destination attributes, and when there is a fixed probability of recalling previously examined destinations, the analysis follows that of Landsberger and Peled (1977). They show that there is a unique sequence of reservation values $\{x_n^*\}$ associated with the sequential search. It is optimal to continue search if the best of the first N-n destinations examined (where N is the total number of destinations), x_n, is less than x_n^*, and it is optimal to terminate the search if $x_n \geq x_n^*$. Note that n is the number of destinations that are left to consider before the search is terminated. The x^*'s are chosen to be equal to the expected return, R_n, from examining one more destination and then continuing in an optimal way. Thus,

$$x_n^* = R_n(x_n^*) = P \{ \int_{-\infty}^{\infty} V_{n-1} [\max(x^*, y)] \, dF(y) \}$$
$$+ (1-P) [\int_{-\infty}^{\infty} V_{n-1} (y) \, dF(y)] - c , \qquad (13)$$

where c is the fixed cost associated with examining an additional destination, P is the probability of successful recall, and $V_{n-1}(y)$ is the expected return from optimal search when y is the best available alternative. The first term on the right-hand

16

side of (13) is the return from optimal search if the best destination to date is still available after the next destination is examined. This occurs with probability P. Otherwise, with probability 1-P, the best of the first N-n will not be available. In this case, the next destination examined will constitute the best destination. This return is represented by the second term on the right-hand side of (13).

In the special case where P=1, recall is perfect, and as Landsberger and Peled show, the optimal reservation value is independent of the time remaining to search. In this case, the solution x^* is determined by setting the marginal return from one more observation equal to the marginal cost, just as in the case with an infinite horizon:

$$\int_{x^*}^{\infty} (x-x^*) \, dF(x) = c.$$

If search costs are zero, $x^* = \infty$, which implies that individuals should examine all possible destinations and then choose the maximum of all observations. This is equivalent to the choice problem analyzed in discrete choice models. In the special case where destinations have utilities with stochastic terms independent identically distributed according to the Gumbel distribution, destination choice will occur according to the standard logit choice model.

Sequential optimization problems allowing recall may therefore be viewed as being more general than simultaneous discrete choice models, since they include them as a special case.

4.1.5 Other Generalizations

Space limitations prohibit a more complete discussion of the broad array of generalizations of optimal search theory that may

17

be found in the literature of economics, operations research, and mathematics. Examples of such generalizations include the following:

1) The number of opportunities is unknown. Rasmussen and Robbins (1975) investigate the decision rule when the number of opportunities is a random variable taken from a known probability density function.

2) When the distributions of destination attractiveness (not necessarily identical across destinations) are known, optimal selection of the order in which destinations should be examined is discussed by Hill and Hordijk (1985) and Maier (1986).

4.2. Generalizations Derivable From a Simultaneous Choice Approach

Optimal search models usually assume that all opportunities are drawn from the same distribution with identical parameters. Discrete choice models, however, allow for a change in the location parameter of the distribution. The utility obtainable from an opportunity is usually divided into a deterministic and a stochastic part. Various types of discrete choice models differ by the distribution assumed for the stochastic part. For example, the iid Gumbel distribution leads to the logit model, and the multivariate normal leads to the probit model.

Since the deterministic part of utility is determined by the characteristics of opportunities and decision makers the discrete choice approach provides an excellent way to account for differences between opportunities and decision makers in the intervening opportunities model. More attractive opportunities will have higher choice probabilities than less attractive ones.

Opportunities located beyond a very attractive one will only have a low chance to be selected. The distance decay in choice probabilities results from the increasing (total) travel costs of search, although this effect might be compensated by higher attractiviteness.

Even the logit model, a rather simple type of discrete choice model, yields a very general pattern of spatial interaction, which is based on a well founded set of behavioral assumptions and on the characteristics of opportunities. Generalizations of the logit model allow for aggregate opportunities (see e.g. Lerman 1975; McFadden 1978) and for some correlation in the stochastic part of utility (see e.g. Ben-Akiva, 1973; Daly and Zachary, 1979; Maier and Fischer, 1985). These more general models can be used to incorporate the effects of agglomeration and specialization in space. Because of the behavioral orientation of discrete choice models they allow for heterogenous decision makers as well. Provided the data are available, an intervening opportunities model based on the discrete choice approach can take into account behavioral differences between socioeconomic groups.

Contrary to the logit model the probit model is able to handle a general variance-covariance-structure in the stochastic part of the utility. Thus it is much more flexible than the logit model. This flexibility is obtained at the cost of computational problems in the multivariate case (Judge et.al., 1980). Since in spatial interaction modeling we are usually dealing with a large number of opportunities, the probit model seems too complex for most purposes.

4.3. The Relationship between Discrete Choice and Optimal Search Models.

At the end of section 4.1 we already have pointed out that the simultaneous choice model can be viewed as a special case of a sequential model with perfect recall. Lerman and Mahmassani (1985) have used the discrete choice rationale to discuss the econometrics of search. However, they have forced discrete choice assumptions upon the search concept ignoring some of its essential features like the optimal reservation value being the expected return of continued search.

There is a more direct and more general correspondence between discrete choice and optimal search models. It will be outlined in the rest of this section. This correspondence seems to be quite promising for making search models operational in empirical applications at an individual level.

The optimal search model we are considering is of the standard type. We assume the individual to know the parameters of the distribution and to search without recall. Generalizing the standard search approach, we allow the opportunities to have different average returns (θ_i). To clarify the relation again we assume these parameters to be expressed in terms of money.

When there are N opportunities and marginal travel costs of search of c_i for going from opportunity i-1 to i, the individual can determine his optimal strategy by backward induction. He will choose opportunity i, given he didn't stop before, if and only if (see equation 3)

$$x_i > y_i \ . \tag{14}$$

Since there are only N opportunities available (see equation 4),

$$y_{N-1} = -c_N + \theta_N \ . \tag{15}$$

In general the expected maximum return of search is defined by

$$y_{i-1} = -c_i + \text{Emax}(x_i, y_i) =$$

$$-c_i + \int_{-\infty}^{y_i} y_i \, dF(x_i) + \int_{y_i}^{\infty} x_i \, dF(x_i) \tag{16}$$

and depends on the distribution of x. Assuming x_i to be logistically distributed with parameters θ_i and μ, i.e.

$$F(x) = 1/(1+e^{\mu(\theta_i - x)}), \tag{17}$$

yields the following result for the parameter y_{i-1}

$$y_{i-1} = (1/\mu)*\log\{\sum_{j=i}^{N} \exp[\mu(\theta_j - \sum_{k=i}^{j} c_k)]\} \quad . \tag{18}$$

This, however, is the inclusive value of the corresponding nested logit model, i.e. the generalization of (12). When substituting (18) and the cumulative density of the logistic distribution into the corresponding choice probabilities of the search model, we get the logit model formulation of choice probabilities:

$$P(i) = \exp[\mu(\theta_i - \sum_{k=1}^{i} c_k)] / \{\sum_{j=1}^{N} \exp[\mu(\theta_j - \sum_{l=1}^{j} c_l)]\} \tag{19}$$

Under these assumptions the sequential search model without recall is equivalent to a simultaneous discrete choice logit model. The distributional assumptions necessary for this result are as restrictive as those of the logit model. The only crucial assumption we had to make was the absence (or neglibility) of search costs proper. In a spatial interaction context, however, this seems to be only a minor restriction since most of the cost involved can be captured by travel costs of search. Moreover, this assumption is important only when the individual can freely choose the sequence in which to check the opportunities. When this sequence is fixed for some reason (e.g. by the spatial distribution of opportunities) search costs proper can also be subsumed under travel costs of search.

It should be mentioned as well that the special treatment of cost in this section was motivated solely by the aim to keep a close formal relationship between search and discrete choice models. In an empirical application cost can easily be incorporated into the function determining the parameter θ and thus have a testable parameter.

5. Summary

In this paper we have argued that the theories of sequential optimization and discrete choice are sufficiently general to allow the derivation of both the intervening opportunities model and the more widely used gravity model. It is demonstrated that the intervening opportunities model in its standard form is equivalent to very restrictive versions of both sequential optimization and discrete choice models. This provides a basis for more general versions of the intervening opportunities model.

Adoption of this more general framework, however, forces the researcher to face more serious estimation problems in applied work. Some of these problems have been addressed by Lerman and Mahmassani (1985). They derive likelihood functions for a variety of sequential optimization problems. It is clear that in many cases the computational burden imposed by the estimation problem will be quite large. Still, relatively small generalizations should be conceptually feasible, thereby allowing the ideas suggested here to be implemented.

Footnote

1) For a discussion of search models without the assumption of risk-neutrality see Hall, Lippman and McCall, 1979.

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